Theory approaches to hadronic parity violation: DDH, EFTs, and all that

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Review
The theory of parity violation in few-nucleon systems

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\textbf{A R T I C L E   I N F O}

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Few-nucleon physics

\textbf{A B S T R A C T}

We review recent progress in the theoretical description of hadronic parity violation in few-nucleon systems. After introducing the different methods that have been used to study parity-violating observables we discuss the available calculations for reactions with up to five nucleons. Particular emphasis is put on effective field theory calculations where they exist, but earlier and complementary approaches are also presented. We hope this review will serve as a guide for those who wish to know what calculations are available and what further calculations need to be completed before we can claim to have a comprehensive picture of parity violation in few nucleon systems.

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Introduction

Models of the NN interactions

Chiral effective field theory

Pionless effective field theory

Conclusion & Outlook
What is hadronic parity violation?

- Parity-violating component in NN interaction
- Suppressed by $\sim 10^{-7}$
- Isolate in pseudoscalar observables $(\vec{\sigma} \cdot \vec{p})$
  - Longitudinal asymmetries
  - Angular asymmetries
  - Spin rotation
  - $\gamma$ circular polarization
  - Anapole moment
Where does it come from?

Weak interaction between quarks
- Standard Model physics
- Mediated by $W, Z$ exchange
- Range $\approx 1/M_W \sim 0.002$ fm
- Well-understood (for free quarks)

So what is the problem?
Why is it complicated?

- No free quarks
- $0.002\text{ fm} \ll r_{\text{nucleon}}$
- Manifestation for quarks bound in nucleon?
- Strong interactions at low energies

Interplay of weak and nonperturbative strong interactions
Why should I care?

- Weak neutral current in hadron sector
- Probe of strong interactions
  - Weak interactions short-ranged
  - Sensitive to quark-quark correlations inside nucleon
  - No need for high-energy probe
  - “Inside-out probe”
- Isospin dependence of interaction strengths?
  \[\Delta I = 1/2\] puzzle (strangeness-changing )?
- Can contribute to T violating observables
Early ideas about NN interaction

NN potential
- Finite range
- Massive particle exchange
- Yukawa: $M \sim 100\,\text{MeV}$

Long-range ($r \gtrsim 1.5\,\text{fm}$): One-pion exchange

- Intermediate distance ($0.7\,\text{fm} \lesssim r \lesssim 1.5\,\text{fm}$): two-pion exchange?
- Short-distance ($r \lesssim 0.7\,\text{fm}$): ?
NN potentials: basics

Symmetry constraints on possible form of NN potential

- Galilei invariance
- C,P,T
- Isospin

\[ V = \sum V_i O_i \]

- Operators \( O_i \): \( 1, \hat{S} \cdot \hat{S}, S_{12}, \hat{L} \cdot \hat{S}, (\hat{L} \cdot \hat{S})^2 \)
- Coefficients \( V_i(r, \hat{p}^2, \hat{L}^2) \): undetermined
Models

Phenomenological models
- Make ansatz for $V_i$
- Long range: one-pion exchange
- $\approx 40 - 50$ parameters
- Fit to NN scattering data
- Interpretation?
- Connection to QCD?

One-boson-exchange models
- Long range: one-pion exchange
- Intermediate and short range: $\sigma, 2\pi, \rho, \omega, \ldots$
- $M_i < 1$ GeV
- Connection to QCD?
PV potential

DDH model

- Single-meson exchange \((\pi^{\pm}, \rho, \omega)\) between two nucleons with one strong and one weak vertex

- Weak interaction encoded in PV meson-nucleon couplings

- Total of 6 (7) couplings

- Estimate weak couplings (quark models, symmetries) ⇒ ranges and “best values/guesses”

Desplanques, Donoghue, Holstein (1980)
DDH potential

\[ V_{\text{DDH}} = i \frac{h_1^i g AM}{\sqrt{2} F_\pi} \left( \frac{\vec{\tau}_1 \times \vec{\tau}_2}{2} \right)_z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[ \frac{\vec{p}_1 - \vec{p}_2}{2M}, w_\pi(r) \right] \\
- g_\rho \left( h_0^\rho \vec{\tau}_1 \cdot \vec{\tau}_2 + h_1^\rho \left( \frac{\vec{\tau}_1 + \vec{\tau}_2}{2} \right)_z + h_2^\rho \frac{3\vec{\tau}_1 \cdot \vec{\tau}_2}{2\sqrt{6}} \right) \\
\times \left( (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \left\{ \frac{\vec{p}_1 - \vec{p}_2}{2M}, w_\rho(r) \right\} + \cdots \right) \\
+ \cdots \\
\]

with

- \( g_M \): strong (PC) meson-nucleon couplings
- \( h_i^M \): weak (PV) meson-nucleon couplings
- \( w_M(r) = \frac{\exp(-m_M r)}{4\pi r} \)
Complications

- Connection to QCD?
- Error estimates?
- Consistency between different parts of calculations?
  - Wave functions
  - Operators
  - External currents (e.m., weak)
  - PC and PV potentials (couplings, particle content, . . .)
  - 3N, 4N, . . . forces?
- “mesons” = “real” mesons? ($1/M_\rho \approx 0.3$ fm)
Experimental constraints

Haxton, Holstein (2013)
Re-analysis of $pp$ scattering at 221 MeV

- Use consistent strong couplings in PC and PV parts

Haxton, Holstein (2013)
Effective field theories

- Low-energy theory for NN interactions
- Effective degrees of freedom: nucleons, pion, . . .
- Symmetry constraints

So what’s new?

- All symmetries of QCD: chiral symmetry
- Only relevant degrees of freedom, not more
- Expansion in ratio of scales $Q/\Lambda < 1$: Power counting
Power counting

Perturbation theory in QED
- Coupling $\alpha \ll 1$
- Expand observables in $\alpha$
- Calculate order by order (Feynman graphs, etc)

Perturbation theory in EFT
- Possibly strong coupling $\mathcal{C} \Rightarrow$ PT in $\mathcal{C}$ not possible
- Identify ratio of scales
  - Typical low-energy (long-distance) scale $E$
  - Underlying high-energy (short-distance) scale $\Lambda$
- Expand observables in ratio $E/\Lambda$
- Calculate order by order (Feynman graphs, etc)
Advantages

- Model independent
- Unified framework
  - PC and PV potentials
  - External currents
  - 2N, 3N, 4N, \ldots
- Systematically improvable
- Theoretical error estimates
Chiral EFT for NN interaction

- Degrees of freedom: nucleons and pions
- Energy scales: $E \approx m_\pi < \Lambda \approx 1 \text{ GeV}$
- Symmetries: Galilei, C, (P), T, chiral (non-trivial)
- Lowest-order PC potential:

$$V_{PC}^{LO} = - \frac{g_A^2}{4F^2} \left( \frac{\vec{q} \cdot \vec{q}}{\vec{q}^2 + m_\pi^2} \right) (\vec{\tau}_1 \cdot \vec{\tau}_2) + C_S + C_T (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

- one-pion exchange
- Contact terms (short-distance operators)
- Low-energy constants $C_S, C_T$
- Systematic corrections possible
  - Higher powers of $q = p_f - p_i$
  - Two-pion exchange
  - ...
Chiral PV NN potential

Leading order:

\[ V_{PV}^{LO} = -i g_A h_{\pi NN}^1 \frac{\sigma_1 + \sigma_2}{q^2 + m_\pi^2} \cdot \vec{q} \left( \vec{\tau}_1 \times \vec{\tau}_2 \right)_z \]

- One-pion exchange \( \propto h_\pi \)
- LO contribution to \( \vec{n}p \to d\gamma \)

Next-to-leading order

- Contact terms
- TPE \( \propto h_\pi \)
- New \( \gamma\pi NN \) contact interaction

Caveat: power counting assumes that \( h_\pi \) is not “small”

Savage, Springer (1998); Kaplan et al. (1999); Zhu et al. (2005); Liu (2007); Song et al. (2011), (2012)
Pionless EFT

- At very low energy: pion-exchange not resolved:
  \[
  \frac{1}{\vec{q}^2 + m^2_\pi} \approx \frac{1}{m^2_\pi} \left( 1 - \frac{\vec{q}^2}{m^2_\pi} + \cdots \right)
  \]

- EFT with only nucleons as degrees of freedom: EFT($\pi$)
- Contact interactions
- Order by number of derivatives
- Lowest-order parity-conserving Lagrangian (partial-wave basis)

\[
\mathcal{L} = N^\dagger (i\partial_0 + \frac{\vec{\nabla}^2}{2M}) N - \frac{1}{8} C_0^{(1S_0)} (N^T \tau_2 \tau_a \sigma_2 N)^\dagger (N^T \tau_2 \tau_a \sigma_2 N) \\
- \frac{1}{8} C_0^{(3S_1)} (N^T \tau_2 \sigma_2 \sigma_i N)^\dagger (N^T \tau_2 \sigma_2 \sigma_i N) + \ldots,
\]

- Connect $C_0^{(1S_0)}$, $C_0^{(3S_1)}$, etc to effective range expansion
Parity violation in EFT(\(\not{\pi}\))

- Parity determined by orbital angular momentum \(L : (-1)^L\)
- Simplest parity-violating interaction: \(L \rightarrow L \pm 1\)
- Leading order: \(S \rightarrow P\) wave transitions

Spin, isospin: 5 different combinations

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Danilov (1965, '71); Zhu et al. (2005); Phillips, MRS, Springer (2009); Girlanda (2008)
Lowest-order parity-violating Lagrangian

Partial wave basis

\[
\mathcal{L}_{PV} = -\left[ C^{(3S_1-1P_1)} \left( N^T \sigma_2 \bar{\sigma} \tau_2 N \right) \dagger \cdot \left( N^T \sigma_2 \tau_2 i\nabla N \right) \right. \\
+ \left. C^{(1S_0-3P_0)}_{(\Delta I=0)} \left( N^T \sigma_2 \tau_2 \bar{\tau} N \right) \dagger \left( N^T \sigma_2 \bar{\sigma} \cdot i\nabla \tau_2 \bar{\tau} N \right) \right] \\
+ \left. C^{(1S_0-3P_0)}_{(\Delta I=1)} \epsilon^{3ab} \left( N^T \sigma_2 \tau_2 \tau^a N \right) \dagger \left( N^T \sigma_2 \bar{\sigma} \cdot \nabla \tau_2 \tau^b N \right) \right] \\
+ \left. C^{(1S_0-3P_0)}_{(\Delta I=2)} \mathcal{I}^{ab} \left( N^T \sigma_2 \tau_2 \tau^a N \right) \dagger \left( N^T \sigma_2 \bar{\sigma} \cdot i\nabla \tau_2 \tau^b N \right) \right] \\
+ \left. C^{(3S_1-3P_1)} \epsilon^{ijk} \left( N^T \sigma_2 \sigma^i \tau_2 N \right) \dagger \left( N^T \sigma_2 \sigma^k \tau_2 \tau^3 \nabla^j N \right) \right] \\
+ h.c.
\]

- Need 5 experimental results to determine LECs

Translation between different formalisms

- DDH operator structure $\rightarrow$ EFT($\not\!$) at low energies
- Can relate DDH and EFT couplings?
- Problems
  - Low-energy limit of PC potentials?
  - Regulator dependence in potential ($\mu_P$): $\frac{\mu_P^2}{4\pi r} e^{-\mu_P r}$
  - Scale dependence in EFT couplings ($\mu$)

Relation between couplings in DDH and EFT scale-dependent

$$h^i_{DDH} = f(\mu_P, \mu) h^i_{EFT}$$
Example: \( nd \) spin rotation

- Hybrid calculation: AV18+UIX (PC) and EFT(\( \pi \))(PV)
- Observable = \( \sum c_n l_n \), \( c_n \): PV couplings

Translated values for \( l_n \) ([\( \mu_p, \mu \] = MeV)

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<th>Hybrid</th>
<th>Translated</th>
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<td>( \mu = 100 )</td>
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<tr>
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<td></td>
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<tr>
<td>1</td>
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<td>-53.2</td>
</tr>
<tr>
<td>9</td>
<td>-6.11</td>
<td>-10.4</td>
</tr>
</tbody>
</table>

Consistent EFT scale-independent

Schiavilla et al. (2008), Grießhammer, MRS, Springer (2012)
PV on the lattice

- Standard Model degrees of freedom (quarks, gluons, . . .)
- Calculating reactions not yet realistic
- Determine PV couplings
- PV quark operators on lattice between 3-quark operators (proton, neutron, etc)

\[ h_\pi = \left( 1.099 \pm 0.505 \text{ (stat.)} \pm^{+0.058}_{-0.064} \text{ (syst.)} \right) \times 10^{-7} \]

- \( m_\pi \sim 389 \text{ MeV, } L \sim 2.5 \text{ fm, } a_s \sim 0.123 \text{ fm} \)
- Connected diagrams only
- Consistent with most model estimates, lower end of DDH “reasonable range”

(a) (b) (c)

Wasem (2012)
Conclusion & Outlook

- Interplay of strong and weak interaction
- At low energies QCD nonperturbative
- Description in effective degrees of freedom
- Phenomenological models
  - Accurate description of NN phase shifts, binding energies,…
  - Consistency?
  - Connection to QCD?
- Effective field theory
  - Model independent
  - Consistency for PC and PV parts
  - Theoretical error estimates
  - Energy range?
- Connection not straightforward
Example: Separation of scales

High school:
\[ \Delta U = mgh \]

College:
\[ \Delta U = -\frac{GMm}{r_f} + \frac{GMm}{r_i} \]

\[ r_i = R, \quad r_f = R + h \Rightarrow \]
\[ \Delta U = m \frac{GM}{R^2} h \frac{R}{R + h} \approx mgh \left( 1 - \frac{h}{R} + \frac{h^2}{R^2} + \cdots \right) \]

for \( h \ll R \)

Expansion parameter \( h/R \)
Chiral Symmetry

- QCD Lagrangian

\[ \mathcal{L}_{QCD} = \bar{q} (i \slashed{D} - \mathcal{M}) q - \frac{1}{4} G_{\mu\nu}, aG^{\mu\nu}_a \]

- Project onto right- and left-handed fields

\[ \mathcal{L}_{QCD} = (\bar{q}_R i \slashed{D} q_R + \bar{q}_L i \slashed{D} q_L - \bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M} q_R) - \frac{1}{4} G_{\mu\nu}, aG^{\mu\nu}_a \]

- For \( \mathcal{M} = 0 \):

\[ \mathcal{L}^0_{QCD} = (\bar{q}_R i \slashed{D} q_R + \bar{q}_L i \slashed{D} q_L) - \frac{1}{4} G_{\mu\nu}, aG^{\mu\nu}_a \]

- Invariant under \( q_R \rightarrow U_R q_R, q_L \rightarrow U_L q_L \)

**SU(2)_L \times SU(2)_R \times U(1)_V** symmetry
Chiral Symmetry II

Spectrum:
- SU(2) multiplets
- No parity doubling
- Pions very light

**Spontaneous** symmetry breaking to SU(2)_V
Pions as Goldstone bosons

**Explicit** symmetry breaking by $m_{u,d} \neq 0$
- Treat $m_{u,d}$ as perturbation