Chiral symmetry and some implications

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Chiral symmetry

Spontaneous symmetry breaking

Implications of chiral symmetry and its breaking
What is chiral symmetry?

Symmetry of Quantum Chromodynamics (QCD)
- $\text{SU}(2)_R \times \text{SU}(2)_L$ (or $\text{SU}(3)_R \times \text{SU}(3)_L$)
- Approximate
- Spontaneously broken
Why is it important?

Symmetries have consequences

- Constraints on spectrum
  - Rotational symmetry in QM \( \Rightarrow \) \((2l + 1)\) degenerate states \(|l, m\rangle\)

- Constraints on interactions
  - Rotational symmetry \(\Rightarrow\) Selection rules for transitions
  - Special relativity \(\Rightarrow\) Maxwell’s equations
  - Gauge invariance \(SU(3) \times SU(2) \times U(1)\) \(\Rightarrow\) Standard Model

Consequences of chiral symmetry

- Multiplets of hadrons
- Pions are very light
- Interactions of pions are small at small energies
- Relations between different low-energy processes
Quantum Chromodynamics

- Theory of strong interactions
- Quarks and gluons
- SU(3) (color) gauge theory
- Lagrangian

\[ \mathcal{L}_{\text{QCD}} = \sum_{f=1}^{6} \bar{q}_f (i \slashed{D} - m_f) q_f - \frac{1}{2} \text{Tr} (G_{\mu \nu} G^{\mu \nu}) \]

- Key features
  - Asymptotic freedom
  - Confinement
Light and heavy quarks

- Light quarks: $u, d, s$:
  
  \[ m_u, m_d, m_s \ll 1 \text{ GeV} \]

- Heavy quarks: $c, b, t$:
  
  \[ 1 \text{ GeV} \ll m_c, m_b, m_t \]

- Masses of observed strongly interacting particles:
  
  \[ M_\rho, M_p, \ldots \sim 1 \text{ GeV} \]

- Exception: $\pi$ mesons:
  
  \[ m_\pi \sim 140 \text{ MeV} \]
Chiral limit

- At low energies: ignore effects from heavy quarks
- Quark model: proton = uud
- $M_p \gg 2m_u + m_d$
- Approximate $m_u = m_d = 0 \text{ MeV}$

Chiral limit = massless light quarks

$$\mathcal{L}_0 = \sum_{f=u,d} \bar{q}_f i \not{D} q_f - \frac{1}{2} \text{Tr} \left( G_{\mu\nu} G^{\mu\nu} \right)$$
Chiral quark fields

- Left- and right-handed quark fields:

\[ q_{f,R} = \frac{1}{2}(1 + \gamma_5)q_f, \quad q_{f,L} = \frac{1}{2}(1 - \gamma_5)q_f \]

- Massless QCD Lagrangian:

\[ \mathcal{L}_0 = \sum_f \left( \bar{q}_{f,R} i\slashed{D} q_{f,R} + \bar{q}_{f,L} i\slashed{D} q_{f,L} \right) - \frac{1}{2} \text{Tr} \left( G_{\mu\nu} G^{\mu\nu} \right) \]

- Collect

\[ q_R = \begin{pmatrix} q_{u,R} \\ q_{d,R} \end{pmatrix}, \quad q_L = \begin{pmatrix} q_{u,L} \\ q_{d,L} \end{pmatrix} \]
**Chiral symmetry**

- **Massless QCD Lagrangian**

\[
\mathcal{L}_0 = \left( \bar{q}_R i \mathbf{
ot} \partial q_R + \bar{q}_L i \mathbf{
ot} \partial q_L \right) - \frac{1}{2} \text{Tr} \left( G_{\mu \nu} G^{\mu \nu} \right)
\]

- Invariance under independent global transformation of right- and left-handed quarks

  \[q_R \rightarrow U_R q_R, \quad q_L \rightarrow U_L q_L\]

- \(U_R, U_L\): independent SU(2) matrices

**Chiral symmetry:**

\[\text{SU}(2)_R \times \text{SU}(2)_L\]
Spontaneous chiral symmetry breaking

- Expect degenerate multiplets of positive and negative parity
- Not observed
- Pions very light compared to all other hadrons

**Spontaneous chiral symmetry breaking:**

$$SU(2)_R \times SU(2)_L \rightarrow SU(2)_V$$

- $SU(2)_V$: isospin symmetry
Spontaneous symmetry breaking

- Symmetry = invariance under transformation
- Transformation defined by group $G$ (translations, rotation, $SU(2)$, ...)
- Assume Lagrangian (underlying laws) invariant under group $G$
- Ground state = lowest-energy state
- Two possibilities for ground state: invariant under
  - Full group $G$ (Wigner-Weyl mode)
  - Only subgroup $H$ (Nambu-Goldstone mode)

Spontaneous symmetry breaking

Ground state is not invariant under full symmetry group of Lagrangian
Spontaneous symmetry breaking: Examples

Ferromagnet

- Underlying laws rotationally invariant
- At $T < T_C$: preferred direction $\Rightarrow$ rotational invariance lost

“Mexican hat” potential

- $V = -a|\phi|^2 + b|\phi|^4$: invariant under $\phi \to e^{i\theta} \phi$
- Ground state “picks” direction
Goldstone bosons

Consequence of spontaneous breaking of continuous symmetry

- Lagrangian invariant under $G$ with $n_G$ generators
- Ground state invariant under $H$ with $n_H < n_G$ generators

Goldstone’s theorem

There are $n_G - n_H$ massless Goldstone bosons
Application to QCD

- Lagrangian invariant under $G = \text{SU}(2)_R \times \text{SU}(2)_L$
  $\Rightarrow n_G = 6$
- Ground state (spectrum) invariant under $\text{SU}(2)_V$
  $\Rightarrow n_H = 3$
- $6 - 3 = 3$ massless Goldstone bosons

Pions = Goldstone bosons of spontaneously broken chiral symmetry
Explicit chiral symmetry breaking

- Finite quark masses

\[ \mathcal{L}_M = - (\bar{q}_R M q_L + \bar{q}_L M q_R) \]

where \( M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \) and \( m_u = m_d = m \)

- No longer invariant under independent \( R \) and \( L \) transformation (\( SU(2)_R \times SU(2)_L \))

- Still invariant under \( SU(2)_V \) (isospin) transformations

- Goldstone bosons acquire mass (pseudo-Goldstone bosons)

\[ m \ll M_{\text{hadron}} \Rightarrow \text{perturbation theory in } m \]
Chiral perturbation theory

Effective theory of strong interactions at $E \ll 1$ GeV

- Degrees of freedom: pions, nucleons, ...
- Low-energy scale: pion (quark) masses and momenta
- High-energy scale: $\Lambda_\chi \approx 4\pi F_\pi \approx 1$ GeV
- Expansion in $\frac{m_\pi, q}{\Lambda_\chi}$: power counting

Pion mass and momentum expansion of observables
Implications of chiral symmetry

- Nonlinear realization of chiral symmetry:

\[ U(x) = \exp \left( i \frac{\phi(x)}{F} \right) = 1 + i \frac{\phi(x)}{F} - \frac{\phi(x)^2}{2F^2} + \ldots \]

\[ \phi = \sum_a \phi a \tau_a = \begin{pmatrix} \pi^0 & \sqrt{2} \pi^+ \\ \sqrt{2} \pi^- & -\pi^0 \end{pmatrix} \]

- Chiral Lagrangian

\[ \mathcal{L} = \mathcal{L}(U) \]

Chiral symmetries of QCD

⇒ Relations between different low-energy processes

Weinberg (67, 79); Gasser, Leutwyler (84)
Lowest-order Lagrangian: Gell-Mann, Oakes, Renner relation

- Leading-order Lagrangian

\[ \mathcal{L}_2 = \frac{F^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{F^2 B^2}{2} \text{Tr}[\mathcal{M}(U + U^\dagger)] \quad (1) \]

\[ = \frac{1}{2} \partial_\mu \phi_a \partial^\mu \phi_a + mB \phi_a \phi_a + O(\phi^4) \quad (2) \]

Pion mass at lowest order

\[ M^2_\pi = 2Bm \]

- Can show \( B \propto \langle \bar{q}q \rangle \) (scalar quark condensate)
  \( \Rightarrow \) Gell-Mann, Oakes, Renner relation
\( \pi N \) coupling and \( g_A \)

- Lowest-order \( \pi N \) Lagrangian

\[
\mathcal{L}_{\pi N}^{\text{int}} = -\bar{\psi} \frac{g_A}{2} \gamma^\mu \gamma^5 u_\mu \psi
\]

\[
= -\bar{\psi} \left( \frac{g_A}{2F} \gamma^\mu \gamma^5 \partial_\mu \vec{\phi} \cdot \vec{\tau} - g_A \gamma^\mu \gamma^5 \frac{\vec{a}_\mu}{2} \right) \psi + \cdots
\]

- Relates \( \pi N \) (strong) and axial-vector (weak) couplings

**Goldberger-Treiman relation**

\[
\frac{g_{\pi N}}{M_N} = \frac{g_A}{F}
\]

- Systematically calculate corrections to relations

Gasser, Sainio, Svarc (88); Goldberger, Treiman (58)
Chiral perturbation theory for $A \geq 2$ nucleons

- Expansion of NN potential
- Leading order: contact terms + one-pion exchange

\[ \pi N \text{ coupling from } L_{\pi N}^{\text{int}} \]
\[ \Rightarrow \text{Relation between NN potential and neutron decay} \]

- Next-to-leading order: contact terms + two-pion exchange
- \ldots

- Relations to $\pi N$ scattering, \ldots
- Extend to three-nucleon, four-nucleon etc potentials
- Consistent potentials and currents
## Chiral expansion of nuclear forces

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<th>Two-nucleon force</th>
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Similarly for currents

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Park et al. (94, 97); Pastore et al. (09, 11)
Applications

- NN scattering
- Deuteron properties
- Quark-mass dependence of NN force
- Three-nucleon forces and observables
- Few-body systems
- Hoyle state in $^{12}$C
- ...
Conclusions and outlook

Chiral symmetry

- Approximate, spontaneously broken symmetry of QCD
- Reflected in hadron spectrum
- Pions as (pseudo-) Goldstone bosons
- Constraints on allowed interactions
- Relations between observables