Nuclear Matrix Elements from Lattice QCD
QCD & Lattice QCD have been around for over 30 years, but...

- Difficult to calculate nonperturbative quantities
- Lattice QCD is the only known way

Lattice R&D

- Fermions, discretization errors, volume effects, quark mass effects....

Theory, Algorithmic, & Computational Power developments make this an emergent technology...
Why do we care?

- **Model-Independent Physics**
  - Nuclear quantities directly from the underlying field theory
  - No models, truncated shell spaces, etc.
  - Fully quantified errors

- **Unknown Quantities**
  - Calculate unknown or poorly constrained quantities

- **The Great Beyond**
  - Discover differences between QCD prediction & experiment
  - Uncover BSM physics?
**QCD & Lattice QCD**

- Interactions between quarks and gluons
- 3 colors, 3 anticolors
- 3 (dynamic) quark flavors
- Nonperturbative in nuclear physics regime
  - Discretize space and time

\[ \Rightarrow \int [d\phi] \rightarrow \prod_n \int_{-\infty}^{\infty} d\phi_n \]
Gauge Configurations

- Monte Carlo evolution weighted by exponential of the action
- Includes:
  - Gluons, gluon interactions
  - Quark loops
Steps to a Lattice Calc.

1. Gauge Configurations
2. Propagator Generation
   - Invert Dirac matrix
   - Calculation on each cfg
   - From source pt, to all other lattice sites
Steps to a Lattice Calc.

1. Gauge Configurations
2. Propagator Generation
3. Quark Contractions
   - Combine 3 props to make a baryon
   - Tie color indices to color singlet
   - Momentum project
Steps to a Lattice Matrix Element

1. Gauge Configurations
2. Propagator Generation
   - Now need 2nd Propagator!
   - Sequential Propagator
   - Direct Propagator
3. Quark Contractions
Correlation Functions

- After spatial sum, left with:

\[ C_A(t) = \sum_{x} \langle A(x, t) | A(0,0) \rangle = \sum_{n} \left| \langle n | A | 0 \rangle \right|^2 e^{-E_n t} \]

\[ \xrightarrow{t \to \infty} \left| \langle a | A | 0 \rangle \right|^2 e^{-E_a t} \]
Correlation Functions

After spatial sum, left with:

\[
C_A(t) = \sum_x \langle A(x, t) | A(0,0) \rangle = \sum_n \left| \langle n | A | 0 \rangle \right|^2 e^{-E_n t}
\]

\[
\xrightarrow{t \to \infty} \left| \langle a | A | 0 \rangle \right|^2 e^{-E_a t}
\]

\[
C_{A \rightarrow B}(t) = \sum_{x,y} \langle B(y, t) | O(y, t_0) | A(0,0) \rangle
\]

\[
= \sum_{n,m} \langle 0 | B | n \rangle e^{-E_n (t-t_0)} \langle n | O | m \rangle \langle m | A | 0 \rangle e^{-E_m t_0}
\]

\[
\xrightarrow{t, t_0 \to \infty} \langle 0 | B | b \rangle e^{-E_b (t-t_0)} \langle b | O | a \rangle \langle a | A | 0 \rangle e^{-E_a t_0}
\]
Correlation Functions

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\[ C_{A \to B}(t) = \sum_{x,y} \langle B(y, t) | O(y, t_0) | A(0,0) \rangle \]

\[ = \sum_{n,m} \left| \langle 0 | B | n \rangle \right| e^{-E_n (t-t_0)} \left| \langle n | O | m \rangle \right| \left| \langle m | A | 0 \rangle \right| e^{-E_m t_0} \]

\[ \lim_{t, t_0 \to \infty} \langle 0 | B | b \rangle e^{-E_b (t-t_0)} \langle b | O | a \rangle \langle a | A | 0 \rangle e^{-E_a t_0} \]
Matrix Element

- Ratio of correlation functions, pictorially:

\[
R = \frac{\langle 0 | A | a \rangle e^{-E_a(t-t_{op})} \langle a | O | a \rangle \langle a | A | 0 \rangle e^{-E_{at_{op}}}}{\langle a | A | 0 \rangle^2 e^{-E_a t}}
\]

\[= \langle a | O | a \rangle \]

\(t, t_{op} \) Large
What can we do with this?

A (nowhere near exhaustive) list:

- **Well-Known Quantities**
  - Nucleon $g_A$
  - Nucleon E&M Quantities

- **Precision Corrections**
  - Muon $g-2$ QCD Correction

- **Unknown Quantities**
  - Dark Matter Interactions
  - Nuclear Parity Violation
Nucleon $g_A$

$$\langle N | \bar{q} \gamma^\mu \gamma^5 \tau^a q | N \rangle$$

- Precision Test of:
  - LQCD
  - HBChPT
- Lack of pion mass sensitivity until...
- ...nearly the physical point.
- arXiv:1112.2435
Nucleon Vector Form Factors

\[ \langle N | \bar{q} \gamma^{\mu} \tau^{a} q | N \rangle \]

- Disconnected pieces
- \( M_{\pi} \sim 300 \text{ MeV} \)
- Magnetic Form Factor
  - Good Agreement
- Electric Form Factor
  - Off a Little
  - Pion Mass Sensitive?
- arXiv:1111.5960
Neutron Transverse Charge Density

\[ \rho_T(b) = \int \frac{dQ}{2\pi} \left[ Q \cdot J_0(bQ)F_1(Q^2) + \sin(\phi) \frac{Q^2}{M_N} \cdot J_1(bQ)F_2(Q^2) \right] \]
Muon $g-2$

$$a_{\mu}^{hvp} = \alpha^2 \int_0^\infty \frac{dQ}{Q} \frac{W(Q^2 / m_{\mu}^2)}{\Pi(Q^2)}$$

$$(Q_\mu Q_v - Q^2 \delta_{\mu v}) \Pi(Q^2) = \int d^4 x e^{iQ \cdot x} \langle \Omega | J_\mu (x) J_v (0) | \Omega \rangle$$

- QED & Expt. Disagree at 3σ
- Dominant QED error due to hadronic correction
- Ken Wilson Lattice Award 2011
Non-(V-A) interactions

- Precision neutron beta decay experiments
- Interact with nucleon via scalar & tensor couplings
- Combine w/ Expt to reduce parameter space.

$arXiv:1112.2435$
Neutral current PV poorly understood
PV interaction ~ 0.002 fm
PV NN force dominated by long-range interactions
- meson exchange models
- weak physics encapsulated in weak vertex
PV signal is dwarfed by QCD: $\Theta(10^{-7})$
Extracting $h_{\pi NN}$

- Experimental
  - NPDGamma
- Theory (models):
  - Quark Model
  - Chiral Solitons
  - QCD Sum Rules
- Lattice QCD needs to step up...
Avoid quark loop contributions, use N* interpolator

\[ m_\pi = 0.06901 \pm 0.00004^{+0.00016}_{-0.00019} \]
\[ m_p = 0.20489 \pm 0.00040^{+0.00049}_{-0.00064} \]
\[ m_{N\pi} = 0.27882 \pm 0.00107^{+0.00203}_{-0.00114} \]
Quark operators known at W, Z scale

\[ L_{PV} = -\frac{G_F \sin^2(\theta_W)}{3\sqrt{2}} \sum_i C_i(\lambda, m_c, m_b) \theta_i \]
The Weak Operator

- Three ways to put together:
  - Connected:
    - Most straightforward
  - Quark Loop:
    - Require point-to-all propagator at ops t
  - Disconnected:
    - Zero in isospin limit
First calculation
Quark Loop Contributions
Improved contraction code
Better Nπ state
Physical pion mass


\[ h^{1}_{\pi NN}(\theta_{\text{con}}) = (1.099 \pm 0.505^{+0.058}_{-0.064}) \times 10^{-7} \]
Take-Away Points:

- Lattice QCD is emerging from decades of R&D to begin making precise determinations of nuclear observables.
- Next generation of computers will achieve physical point calculations.
- Problems remaining: disconnected diagrams, multiple nucleon contractions, baryon noise...
- Lets get to work....