Finite elements analysis of the magnetic field in the presence of the Super Mirror Polarizer and the guide coils for the NPDGamma experiment.

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1. Introduction.

To polarize the neutron beam, the NPDG experiment at the SNS is considering the use of a Super Mirror Polarizer (SMP) in lieu of the $^3$He spin filter used at LANSCE. This report deals with the magnetic field needed for the operation of this polarizer and its impact on the experiment concerning magnetic field gradients. The SMP is followed by a Spin Flipper and the Hydrogen target.

There are a few stringent requirements for the design consideration of the Super Mirror:

- The magnetic field inside the super mirror has to be 300 G or larger to transmit with high efficiency only the neutrons with a given orientation of the spin and to block the neutrons with the opposite spin orientation.
- The magnetic field gradient after the Spin Flipper has to be smaller then 1 mG/cm.
- The field uniformity inside the Spin Flipper has to be better then 1%.

The Super Mirror is made from parallel layers of Fe and Si. These layers are perpendicular to the neutron beam axis. The Fe layer is ferromagnetic while the Si layer is not. The most convenient way to magnetize the Fe layers is to place parallel stacks of NdFeB permanent magnets at equal distances along the neutron beam direction and on the right and left (of the beam) sides of the Super Mirror. The remanent supermirror transmission neutron polarization introduces spin selection into the experimental set-ups in a comfortable way since it does not alter the beam path.

All the components of the NPDGamma experiment (Super Mirror, Spin Flipper and Hydrogen Target) are inside a magnetic field generated by two pairs of guide coils. The magnetic field gradient along the vertical direction and inside the volume of the neutron beam, Spin Flipper and Hydrogen Target has to be smaller then 1 mG/cm to minimize the force acting on the polarized neutrons due to the vertical gradient of the field.

This work is concerned with the calculation of the magnetic field gradients along the neutron beam line from the exit of the Super Mirror to the center of the guide coils. The change in the magnetic field when the Super Mirror is placed inside a magnetic shield is examined.

In section 2 we present the geometry of the Super Mirror and magnets. The first calculations of the magnetic field are presented in section 3 where we consider the Super Mirror and 12 pairs of NdFeB stacks (in the absence of the guide coils). A
more complex model that consists of a second magnet placed around the first one is considered in section 4. The reasons of considering this model and its advantage are explained. This model is placed inside the guide coils in section 5. The final magnetic fields and their gradients are calculated in the same section.

2. The geometry.

The remnant magnetic field inside the Super Mirror is provided by a set of 48 stacks of NdFeB magnets. The “bricks” of each stack are rectangular NdFeB (N352) magnet (figure 1).

![Figure 1. This is the geometry of the rectangular NdFeB (N352) of the stack. The remnant magnetic field is $Br > 1.17 \, T$. The magnet is coated with a protective layer of Ni and Cu.](image)

The initial calculations have considered 12 pairs of NdFeB stacks. The dimensions of the NdFeB stacks that make up the magnet (named Magnet1) and the relative distance these stacks are presented in figure 2A and 2B.
Figure 2A. The geometry of the Super Mirror Polarizer is presented in a plane perpendicular to the neutron beam axis (OZ) and passing through the middle of the SMP. The dimensions of the NdFeB magnets (blue) and of the Iron plates (green) are in centimeters. The neutron beam axis is perpendicular to the page.

Figure 2B. These are the dimensions and position of the Super Mirror Polarizer (SMP) sandwiched between two Iron plates (0.8 cm thick) and 12 stacks of NdFeB. All the dimensions are in centimeters. The direction of the neutron beam line is indicated by the red arrow.

The mesh was built by extruding the first layer (base plane) along the OY direction. The top and bottom Iron plates are built in the third and fifth layer of the mesh. Some of the extrusion layers are presented in figures 2C and 2D. Since there are no another coils, the magnetic field of the NdFeB magnets with top and bottom Iron plates is symmetric with respect to the vertical plane YOZ plane (that contains the axis of the neutron beam). The field is also symmetric with respect to the vertical XOY plane passing through the center of the Super Mirror Polarizer.
Figure 2C. This is a horizontal cross section parallel with the axis of the beam and passing through the Iron Plate of the Super Mirror.

Figure 2D. This is a horizontal cross section through the 6 pillars of NdFeB magnets located in the X<0, Z<0 quadrant of the model. In the vertical direction (OY), the mesh extends from Y=-400 cm to Y=0 cm. The magnetic field is normal to the top of the mesh (Y=0 cm) and is tangent to the YOX and ZOX planes. The height of the Super Mirror is 14 cm. The thickness of the Iron Plates is 0.8 cm long and extend between Z=-25 and 25 cm. This is only a quarter of the mesh. After the model was solved, the entire model was built by reflection in the ZOY and XOY planes.

There are 24 pillars on the right lateral side and 24 pillars on the left lateral side of the neutron beam axis. The direction of the magnetization in NdFeB is along the long axis of the pillars (OY). The height of the pillars is 140 mm. The base of each pillar in the horizontal cross section of the figure is 1.4 cm wide (along the OX axis) and 2 cm wide (along the OZ axis of the beam).
The magnetization curves of NdFeB and Iron (ARMCO) used in Tosca (Opera 3D) are presented in figures 3A and 3B.

**Figures 3(A, B).** The magnetization curves for NdFeB and ARMCO are presented in figure 3A and figure 3B respectively.

Inside the Super Mirror the magnetic field has to be bigger then 300 Gauss to insure the high polarization efficiency of this device (figures 3C and 3D). As can be seen in these pictures the reflectivity of the neutrons with spin up and spin down depends on the magnetization of the layers inside the Super-mirror.
3. The magnetic field inside the Super Mirror (without the guide coils and without the magnetic shield of the cave).

The center of the Super Mirror coincides with the center of symmetry of the system and with the origin of the system of coordinates. The boundaries of all the magnet and its Iron plates along the OZ, OX and OY axis are at Z=+/− 25 cm, X=+/− 8.0 cm and Y=+/− 7.8 cm respectively. There are no external fields and no coils. The top and bottom of the Super Mirror are two plates of Iron (ARMCO) 8 mm thick. The vertical axis (OY) is perpendicular to these plates.

The remnant magnetization of the NdFeB magnets is along the vertical direction along the long axis of the pillars and it can be either up (along +OY) or down (along −OY direction). If the direction of the remnant magnetization inside the NdFeB stacks is along −OY direction then the magnetic field By inside the volume of the neutron beam (-5 cm <X< 5 cm, -6 cm <Y< 6 cm) is positive. The magnetic field was calculated inside the volume of the beam considered a rectangular box with cross sectional area 10 cm X 12 cm. In figures 4 (A, B) the total magnetic field is...
calculated in two vertical planes perpendicular to the OZ (beam) axis at Z=0 cm (center of the Super Mirror) and Z=20 cm, and inside the volume of the beam.

In figures 4 (C and D) the By component of the magnetic field is calculated in two vertical planes parallel with the axis of the beam, that intersect the OX axis at X=0 cm and X=5 cm. The entrance and exit faces of the Super Mirror are located at Z=+/- 25 cm.
The By component of the magnetic field is calculated in two vertical planes parallel with the axis of the beam that intersect the OX axis at X=0 cm and X=5 cm (figures C and D respectively).

The total magnetic field calculated along directions parallel with OZ axis is presented in figure 5 (A) for Y=0 cm (in the horizontal XOZ plane) and Y=5 cm (closer to the top iron plate).

Figure 5 (A, B). The magnetic field |B| is calculated along directions parallel with the OZ beam axis in the horizontal XOZ plane (X=0 ...5 cm) (in figure 5A) and in a plane parallel with XOZ but intersecting the OY axis at Y=5 cm.

In general magnetic field gradient along the horizontal direction (dBy/dx) is smaller then the field gradient along the vertical direction (dBy/dy). The magnetic field inside the Super Mirror is 350 G. Both gradients are between 9 and 14 G/cm at 5 cm distance downstream from the exit of the Super Mirror and decrease quickly to less then 3 G/cm oat 15 cm from this exit. In figures 6 (C, D, E, F) we present the magnetic field gradient dBy/dy in four vertical planes perpendicular to the neutron beam axis (OZ). The distances between these planes and the center of the Super Mirror are 60 cm, 80 cm, 100 cm and 120 cm. These three dimensional plots are function of X and Y of the field gradient inside the volume of the neutron beam (-6 cm <Y < 6 cm and -5 cm <X< 5 cm).
Figure 6 (C, D, E, F). The magnetic field gradient $\frac{dB_y}{dY}$ (mG/cm) in four vertical planes (at $Z=-120$ cm, -100 cm, -80 cm and -60 cm) are presented as a function of $X$ and $Y$ (inside the neutron beam volume, $|X|<5$ cm and $|Y|<6$ cm). The magnetic field in the center of the Super Mirror ($B_0$) is about 360 Gauss.
For a magnetic field $B_0=360$ Gauss the magnetic field gradient inside the neutron beam volume is bigger than 2 mG/cm if the distance to the center of the Super Mirror polarizer is smaller than 120 cm.

To find the smallest distance where the Spin Flipper can be located relative to the center of the Super Mirror, we present (in figure 7(A, B)) the average magnetic field gradients $|dBy/dy|$ and $|dBy/dx|$ computed in vertical planes perpendicular to the OZ axis. The magnetic field gradients have been computed only inside the limits of the neutron beam volume ($-5 \text{ cm} < X < 5\text{ cm}$ and $-6 \text{ cm} < Y < 6 \text{ cm}$). The points of intersection of the vertical with the OZ axis are between $Z=-200 \text{ cm}$ and $Z=0 \text{ cm}$. The center of the Super Mirror is at $Z=0 \text{ cm}$. The magnetic field gradient $<dBy/dy>$ has a maximum at the entrance or exit of the Super Mirror ($Z=-25 \text{ cm}$ or $Z=25 \text{ cm}$ respectively).

Figure 7(A, B). The magnetic field gradients $<|dBy/dy|>$ and $<|dBy/dx|>$ are computed along the beam direction from $Z=-200 \text{ cm}$ to $Z=0 \text{ cm}$ (the center of the Super Mirror). The results obtained using two increments along the OY vertical axis ($DY=0.1 \text{ cm}$ and $0.2 \text{ cm}$) for the calculation of the gradient field are presented in figure A.
Figure 7 (C, D) The same plane average magnetic field gradients as in figure 7(A, B) are presented in these figures only outside the Super Mirror at a distance greater then 45 cm from the center of the Super Mirror (20 cm from the exit or entrance faces).

We conclude that the local magnetic field gradient $dBy/dy$ on the neutron beam axis is smaller then 1 mG/cm only if the distance to the center of the Super Mirror is bigger then 110 cm. Also the magnitude of the magnetic field is bigger then 1 Gauss everywhere from the Super Mirror exit to about 100 cm distance from its center. The magnetic fringe fields are therefore big for this configuration of magnets. To decrease these magnetic fields along the neutron axis we will investigate another model that contains a second magnet (named Magnet2) placed around the first magnet (Magnet1). For the first calculations we consider that the centers of both magnets coincide with the center of the Super Mirror. We will show that this system of magnets can reduce both the magnitude and the gradient of the magnetic field inside the Spin Flipper. The system of two magnetic dipoles (named Geometry A) is presented in (figure 8 C, D, E).

As can be seen in these figures there are 14 pairs of NdFeB in Magnet1. The first model that was used in our calculations has only 12 pairs of NdFeB stacks inside Magnet1. For this configuration of magnets the magnetic field in the center of the Super Mirror is smaller then 300 G only if the height of the NdFeB stacks in Magnet2 is smaller then 30 cm. Since shorter NdFeB stacks are cheaper, we have considered an increase in the number of NdFeB stacks to 14 pairs without increasing the length of the Super Mirror (50 cm). In his case the magnetic field everywhere inside the Super Mirror is bigger then 320 Gauss. The magnetic field and its gradient will be calculated for this system of two Magnets and the results will be compared with the magnetic field gradient and fields when there is only Magnet1.
Figure 8 (A, B). These are two cross sections through the configuration of magnets named Geometry A. There are 14 pairs of NdFeB stacks for Magnet 1. The cross section through the middle of the Super Mirror and perpendicular to the axis of the neutron beam is presented in figure 8(A). The cross section in the horizontal plane (XOZ) is presented in figures 8 (B). The
dimensions are in cm. We indicate the shim Iron plates (that can be added in practice to adjust the magnetic field outside the super mirror) with the green rectangles placed close to the NdFeB bigger magnets.

The two top and bottom plates for each magnet are made from high purity iron (Armco). The magnets are made from NdFeB. The direction of the magnetization of the smaller magnets (or Magnet 1) is opposite to the direction of magnetization of the bigger magnet (or Magnet 2).

The optimum geometry of Magnet 2 will provide the smallest variation of the magnetic field of the magnets inside the volume of the Spin Flipper and will decrease the magnetic field gradients dB/dy and dB/dx downstream from the exit of the Super Mirror as much as possible. To justify the necessity of 14 pairs of NdFeB inside Magnet2, we will present first the results obtained with a 12 pairs of NdFeB stacks inside Magnet 1 with thickness T1=1.4 cm and width W1=2 cm. Keeping this geometry of Magnet1 unchanged we will vary the following dimensions of Magnet 2 (in figure 8B):

a) The distance Lx (measured in the OX direction from the neutron beam to the lateral face of the NdFeB stacks).

b) The distance DV (measured in the OX direction from the axis of the beam to the edge of the Iron Plates).

c) The thickness T2 (cm), the weight W2 (cm) and the height H2 of the NdFeB stacks inside Magnet2.

Figure 8(C). This is a 3 dimensional picture of the two magnets (in Geometry A) seen in Post-Processor.
Keeping the distance $L_x=12.7\text{ cm}$ and $D_V=13.4\text{ cm}$ and changing only the thickness $T_2$ of the NdFeB stacks of Magnet 2, one can change significantly the field outside the Super Mirror if $T_2>1.2\text{ cm}$ (figure 8D). The fast decrease of the magnetic field occur closer to the Super Mirror if the thickness is $T_2=1.5\text{ cm}$ then when it is $T_2=1.4\text{ cm}$. If we outer edge of the NdFeB stacks in Magnet 2 coincide with the outer edge of the Iron plates ($L_x=D_V=13.4\text{ cm}$) then the magnetic field profile along the same direction (OZ) (in figure 8E) remains about the same as in the case when these edges do not coincide (figure 8D). The field $B(0,0,Z)$ calculated along the OZ axis was divided with the field in the center of the Super Mirror $B_0$.

Figure 8 (D, E). The ratio between the magnetic field along the OZ axis $B(0,0,Z)$ and the magnetic field in the center of the Super Mirror Polarizer $B(0,0,0)$ (for different thicknesses $T_2$ of the 30 cm high NdFeB stacks in Magnet 2, and for $L_x=12.7\text{ cm}$) is presented in figure 8(D). The same ratio was calculated for a distance $L_x$ equal with the half width of the top and bottom iron plates ($L_x=D_V=13.4\text{ cm}$) in figure 8(E).

The magnetic field of Magnet 2 starts to cancel the magnetic field of Magnet 1 when the thickness of the NdFeB stacks ($T_2$) in Magnet 2 is bigger then $1.3\text{ cm}$ (for a height of these stacks is $H_2=30\text{ cm}$). This happens when the total volume of the NdFeB stacks in Magnet 2 becomes equal with the total volume of NdFeB stacks in Magnet 1:

$$12 \times (2\text{cm}) \times T_2 H_2 = 24 \times (2\text{cm}) \times T_1 H_1$$

Using this relation with $T_1=1.4\text{ cm}$, $H_1=14\text{ cm}$ and $H_2=30\text{ cm}$ one gets $T_2=1.3\text{ cm}$. 

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Figure 8 (D, E). The ratio between the magnetic field along the OZ axis $B(0,0,Z)$ and the magnetic field in the center of the Super Mirror Polarizer $B(0,0,0)$ (for different thicknesses $T_2$ of the 30 cm high NdFeB stacks in Magnet 2, and for $L_x=12.7\text{ cm}$) is presented in figure 8(D). The same ratio was calculated for a distance $L_x$ equal with the half width of the top and bottom iron plates ($L_x=D_V=13.4\text{ cm}$) in figure 8(E).
If the height of the NdFeB in the second magnet is smaller then H\textsubscript{2} < 30 cm then the magnetic field in some regions inside the Super Mirror (B\textsubscript{0}) also decreases bellow 300 G (which is the minimum value of the magnetic field required inside the Super Mirror).

The magnetic field gradient \(\frac{d\text{By}}{dy}\) divided with the field in the center of the Super Mirror (B\textsubscript{0}) is presented in figure 8 (F).

The effect of shim Iron plates (with different thicknesses) placed on the vertical external surface of the NdFeB stacks (like the green rectangles in figure 8B) when the dimensions of the second Magnet are DV=13.4 cm, Lx=12.7 cm, T\textsubscript{2}=1.5 cm, H\textsubscript{2}=30 cm can be seen in figure 8 (G).

\[ \frac{d\text{By}}{dy} \text{ divided with } B_0 \]

Figure 8 (F and G). The magnetic field gradient \(\frac{d\text{By}}{dy}\) divided with the field in the center of the Super Mirror (B\textsubscript{0}) is represented along the OZ axis for different thickness of the NdFeB stacks (T\textsubscript{2}) in Magnet 2 (figure F).

Shim Iron rectangular plates with thickness smaller then 0.4 cm can be used to adjust the magnetic field outside the Super Mirror along its axis (OZ). The change in the field is obvious for a distance bigger then 50 cm from its center (figure G).

To increase the magnetic field in the center of the Super Mirror (B\textsubscript{0}), one can increase the distance Lx or the height H\textsubscript{2} of the NdFeB stacks in Magnet 2. The magnetic field along the OZ axis divided with the field B\textsubscript{0} in the center of the Super Mirror is calculated for increasing heights H\textsubscript{2} from 40 cm to 130 cm in figure 8 (H). If the height of these stacks decreases bellow 40 cm then the field in the center of the magnet becomes smaller then 300 G. The magnetic field at Z=-24 cm (and 24 cm) (inside Magnet1) is smaller then 300 G, for all values of the height H\textsubscript{2} considered. The magnetic field at Z= -23 and +23 cm inside the Super Mirror is bigger then 300 G only if H\textsubscript{2}> 110 cm (figure 8(I)). The solution to this problem is to increase the number of NdFeB.
magnets (per unit length along OZ axis) keeping the same dimensions of the Super Mirror, or to find another geometry of the Iron plates in Magnet 2 that increases the magnetic field in the center of the Super Mirror while the magnetic fields of Magnet 1 and Magnet 2 still cancel outside the Super Mirror and along the OZ axis of the beam. Both these directions will be followed in this work.

Figure 8 (H, I). The ratio of the magnetic field along the axis of symmetry the Super Mirror (OZ) divided with the magnetic field B(0, 0, 0) in the center of the super mirror, is plotted in figure 8(H) for different heights of the NdFeB stacks (H2) in Magnet 2. The magnetic field at Z=23 cm, 24 cm and 25 cm is presented in figure 8(I) versus the height H2 of the NdFeB stacks in Magnet 2.

The relative field decreases quickly at Z about 60 cm and remains smaller than the relative field of Magnet 1 only if the height of the NdFeB stacks is smaller than 50 cm. When the height H2 is smaller than 40 cm the field in the center B0 becomes smaller than 300 G.

The magnetic field gradient <dBy/dy> divided with the field in the center of the Super Mirror (B0) can be seen in figure 8 (J). The magnetic field gradient dB/dZ divided with the field in the center (B0) along the OZ axis is presented in figure 8(K). This field gradient has to be as small as possible inside the volume of the Spin Flipper. For a field B0=350 G, this field gradient is smaller than 35 mG/cm if Z>61 cm and is smaller than 3.5 mG/cm if Z>175 cm.
Figure 8(J, K). The average magnetic field gradient $<\text{d}B/\text{d}y>$ divided with the field in the center $B_0$ (figure J) and the total field gradient $\text{d}B/\text{d}Z /B_0$ (figure K) are calculated along the OZ axis.

The magnetic field in the center of the Super Mirror changes slowly when the distance $(2*\text{Lx})$ between the pairs of NdFeB stacks in Magnet 2 increases. For example if this distance is varied with two centimeters, the magnetic field in the center of the Super Mirror remains about the same (figure 8(L) and 8(M)).
The ratio of the magnetic field along the OZ axis of the beam and the magnetic field in the center of the Super Mirror is calculated for different distances (DV) between the opposite NdFeB stacks in Magnet 2 (figure 8(L)). The height of these stacks is 60 cm and the field in the center of the Super Mirror is bigger than 320 (G). In the absence of Magnet 2 the field in the center of the Super Mirror is about 360 (G).

The average magnetic field gradient $<dBy/dy>$ (divided with the field in the center $B_0$) is computed in vertical planes perpendicular to the axis of the neutron beam is done for 60 cm high NdFeB stacks (figure 8(M)) for the same range of distances $DV=Lx$.

In figure 8(L, M) the field $B_0$ in the center of the Super Mirror is above 320 (G). At a distance bigger than 110 cm, the magnetic field and the field gradient $dBy/dy$ are about equal with the field and gradient of Magnet 1 in the absence of Magnet 2.
The calculations were also done for different positions of the NdFeB stacks along the OZ direction. There is not a significant change of the magnetic field distribution along the OZ outside the Super-Mirror, when the distribution of the NdFeB stacks in Magnet 2 is changed (while their height, width and thickness remain the same and the geometry of the Iron plate is not changed).

To increase the magnetic field inside the Super Mirror above 300 Gauss even for NdFeB stacks (in Magnet 2) shorter than 30 cm high, we increase the number of NdFeB pairs (in Magnet 1) to 14. The thickness, width and height of these stacks are T1=2.2 cm W1=1.858 cm and H1=14 cm respectively. There are 6 pairs of NdFeB stacks in Magnet2 with thickness T2, width W2=W1 and height H2=26 cm. As a result of the increase in the thickness (T2) of the NdFeB stacks in Magnet2, the magnetic fields of the two magnets (outside the Super Mirror) almost cancel each other when T2 is about 3.1 cm (figures 9A and 9B). The average magnetic field gradient along the vertical direction $<dBy/dy>$ becomes smaller than 1 mG/cm for any distance greater than 75 cm from the center of the Super Mirror.

![Figure 9 (A, B). The relative magnetic field $B(0, 0, Z)/B_0$ is calculated for different thicknesses of the NdFeB stacks (Magnet2) in figure 9(A). For comparison this ratio is presented on the same plot in the absence of Magnet2. The magnetic field gradient averaged on vertical planes that are perpendicular to the axis of the beam is calculated for the same three thicknesses in figure 9(B).](image)

Three dimensional pictures of the magnetic field gradient $<dBy/dy>/B_0$ (in vertical planes perpendicular to OZ) are presented in figure 10 (A, B, C, D). The planes intersect this axis in Z=-60 cm, Z=-80 cm, Z=-100 cm and Z=-120 cm.
Figure 10 (A, B, C, D). The magnetic field gradient $<\text{dBy/dy}>$ (mG/cm) is plotted versus $X$ and $Y$ coordinates in vertical planes perpendicular to the axis of the beam ($Z=\text{-60 cm, -80 cm, -100 cm and -120 cm}$).
We conclude that the entrance face of the Spin Flipper (40 cm long) can be placed at a minimum distance equal with 80 cm from the center of the Super Mirror in the absence of the coils. The magnetic field in the center of the Super Mirror is about 415 Gauss for 14 pairs of NdFeB stacks in Magnet1 (T1=2.2 cm thick, W1=1.858 cm wide and H1=14 cm height) and 6 pairs of NdFeB stacks (T2=3.2 cm thick, W2=1.858 cm wide and H2=26 cm height).

If the thickness of the NdFeB stacks in Magnet 1 is reduced to T1=2 cm then the thickness of the NdFeB stacks in Magnet 2 (26 cm height) has to be between 2.7 cm and 2.8 cm (figure 11 A and figure 11 B). For T2=2.7 cm, the magnetic field in the center B0 decreases to 373 Gauss. Everywhere inside the Super Mirror the field is bigger then 300 Gauss.

![Figure 11 (A, B). The magnetic field along the OZ axis B(0, 0, Z) divided with the field in the center of the Super Mirror (B0) is plotted versus Z (in figure 11A). The average magnetic field gradient (computed on vertical planes perpendicular to the OZ axis) is presented as a function of the distance (Z) from the center of the Super Mirror, for two thicknesses (T2) of the NdFeB stacks in Magnet2 (figure 11(B)).](image)

For this geometry of the magnets, if the thickness of the NdFeB in Magnet2 (T2) is between 2.71 cm and 2.81 cm then the field gradient dBy/dy is smaller 1 (mG/ cm) if the distance (measured along the OZ axis) to the center of the Super Mirror is bigger then 85 cm.
The relative magnetic field downstream from the Super Mirror and along the OZ axis depends very weakly on the width (2*DV) of the Iron Plate in Magnet2 (figure 12 A and 12 B). Therefore in practice it is better to consider a smaller width of the Iron Plates for Magnet2. In the next calculations we consider that this width is equal with 2*DV=23.2 cm.

Figure 12 (A, B) The magnetic field along the OZ divided with the magnetic field in the center of the Super Mirror (B0) is calculated for three widths (along OX) of the Iron Plate in Magnet 2 (given by DV and Lx=DV-0.2 cm) (figure 12A). The average magnetic field gradient (in vertical planes perpendicular to OZ axis) is calculated at different positions outside the Super Mirror (between Z=25 cm and Z=200 cm) in figure 12(B).

To maintain the cancellation of the fringe fields of the two magnets one has to keep the total volume of the NdFeB stacks in Magnet2 constant when their high is decreased. Therefore the thickness of these stacks was varied such that:

\[ T_2 = \frac{T_1 \cdot (7 \cdot H_1)}{(3 \cdot H_2)} \]

T1 and H1 are the thickness and the height of the NdFeB stacks in Magnet1.

Once the height “H2” decreases the field in the center of the Super Mirror also decreases (figure 13 A, B). If H2 decreases bellow 20 cm, the ratio B(0, 0, Z)/ B0 increases everywhere along the OZ axis from Z=0 to Z=200 cm. To keep the magnetic field in all the volume of the Super Mirror (-24 cm < Z< 24 cm) above 300 Gauss, the height of the NdFeB stacks in Magnet 2 should not be made smaller then 25 cm (if there are 14 pairs of NdFeB stacks in Magnet1, of height H1=14 cm, thickness T1=2 cm and width W1=1.858 cm respectively).
Figure 13 (A, B) The magnetic field along the OZ axis divided with the field (B0) in the center of the Super Mirror is presented in figure 13(A) for four heights of the NdFeB stacks in Magnet2. The magnetic field along the OZ axis (B(0, 0, Z)) is calculated inside the Super Mirror for different heights of the NdFeB stacks (H2) in magnet2 (figure 13(B)).

We notice that the relative magnetic field B(0, 0, Z)/B0 has a non-zero slope for |Z|>120 cm. We would like to decrease the magnetic field gradient in this region (after the Spin Flipper) even more. One solution is to translate the center of Magnet2 (of coordinates (0, 0, Z2)) along the OZ axis while the center of Magnet1 (namely (0, 0, Z1)) is not moved. We define DZ to be the difference Z2-Z1. The average magnetic field gradient <dBy/dy> and the relative field B(0, 0, Z)/B0 are presented in figures 14 (A, B).
Figure 14(A, B): The magnetic field divided with the field in the center and the magnetic field gradient (outside the Super Mirror, Z>25 cm) are calculated for five distances DZ=Z2-Z1. Z1 and Z2 define the position of the center for Magnet1 and Magnet2 respectively.

Therefore if the Magnet2 is displaced with 0.5 cm upstream relative to the center of Magnet1 then the magnetic field gradient gets smaller and the magnetic field \(B(0, 0, Z)\) divided with the field in the center of the Super Mirror gets smaller only at a distance bigger then 110 cm from the center of the Super Mirror.

4. The calculation of the magnetic field inside the Super Mirror in the presence of the guide coils (and no magnetic shield).

By convention we choose the center of the guide coils in the center of the coordinate system. The center of the super mirror defined by the coordinates (0, 0, Zc) is located upstream along the OZ axis. In the absence of the shield there is no need to consider the correction coil and only the two pairs of guide coils will be used to get advantage of the symmetry of the field in the horizontal XOZ plane.

First we consider only “Magnet1” placed between the pair of middle coils. The distance between the center of the Super Mirror and the center of the coils will be varied and the magnetic field gradients \(\frac{dBy}{dy}\) and \(\frac{dBy}{dx}\) will be calculated for all these distances.
The position of the guide coils will not be changed (figure 15). The distance between the electrical centers of the middle pair of coils is 50 cm. The outside (or upper) guide coils are 221 cm apart. The electric current in the middle pair and upper pair of coils is 418.9 (A) and 1122 (A) respectively.

Figure 15. The center of the two pairs of guide coils is placed at Z=0 cm. The Super Mirror polarizer is centered at Zc<0 and Xc=Yc=0 cm. The location of the Super Mirror (Zc) was changed and the magnetic field gradient was calculated for each location. The magnetic field gradients \( \frac{dB_y}{dy} \) and \( \frac{dB_y}{dx} \) are calculated for different positions (Zc) of the Super Mirror along the OZ axis.

The absolute value of the magnetic field gradients \( |\frac{dB_y}{dy}| \) and \( |\frac{dB_y}{dx}| \) (G/cm) were calculated in vertical planes perpendicular to the axis of the neutron beam (figure 15 (A, B). The magnetic field By was calculated inside the boundaries of the neutron beam -5 cm<\(X<5\) cm, -6 cm \(<Y<6\) cm.
The average of the magnetic field gradients $|\frac{dBy}{dy}|$ and $|\frac{dBy}{dx}|$ is calculated in vertical planes perpendicular to the beam (OZ) axis. These planes intersect the OZ axis at Z0 (cm). The center of the coils is in the center of the coordinate system (Z=0). The center of the Super Mirror Polarizer is at Xc=Yc=0 and Zc (cm). The field gradients were calculated for each position of the SMP given by Zc (where Zc varies from -100 cm to -200 cm). The field in the center of the coils is B0=10 G.

The most important contribution to the Stern Gerlach steering of the neutrons comes from the field gradient $\frac{dBy}{dY}$. It is important to assure a magnetic field gradient smaller than 1 mG/cm from the center of the Spin Flipper downstream. The magnetic field gradient $\frac{dBy}{dY}$ inside the Spin Flipper (located between Z=-70 cm and Z=-30 cm) can be reduced by increasing the distance (D) between the center of the Super Mirror (50 cm long) and the center of the coils (304 cm long). The results presented in figure 15(A, B) suggest that the field gradient is smaller than 1 mG/cm everywhere inside the volume of the Spin Flipper only if this distance (D) is bigger than 195 cm. This can be seen from the plot of the average magnetic field gradients over the volume of the Spin Flipper versus the distance (figure 16 A, B, C, D).
Figure 16 (A, B, C, D). The volume average magnetic field gradients $<dBy/dy>$ and $<dBy/dx>$ (inside Spin Flipper) are calculated for different distances ($D$) between the center of the Super Mirror and the center of the Guide Coils.

In the following calculations we add the bigger magnet (Magnet2) in our model in addition to the Magnet1 and the Super Mirror (like in figures 8 A, B, C). A three dimensional picture of the Magnets and guide coils is presented in figure 17.
The calculations were done for three positions of the center of the Super Mirror: $Z_c = -130$ cm, -120 cm and -110 cm. The center of the guide coils is in origin $(0, 0, 0)$. For each position of the Super Mirror the thickness ($T_2$) of the NdFeB stacks in Magnet2 was varied between 2.9 cm and 3.5 cm. The magnetic field $B_0$ in the center of the Super Mirror placed in any of these three positions is always bigger than 380 G. This field has a small decrease when the Super Mirror is approached to the center of the coils. The magnetic field uniformity $(B-B_0)/B_0$ (%) is calculated for $Z$ between -85 cm and 20 cm and for $Z$ between -75 cm and 20 cm. The Spin Flipper is located between $Z=-71$ cm and $Z=-30$ cm if the center of the guide coils is at $(0, 0, 0)$.

In the presence of the two magnets and four guide coils, the magnetic field uniformity everywhere inside the Spin Flipper is smaller than 0.6% if the distance between the center of the Super Mirror and the center of the coils is bigger than 120 cm (figure 17 (B, C) and 18 (B, C)). The system of the magnets and Super Mirror is 50 cm long in the neutron beam direction. Therefore on the basis of these results we conclude that the magnetic field uniformity inside the Spin Flipper is smaller than 0.7% for a distance between the exit face of the Super Mirror and the entrance face of the Spin Flipper not smaller than 25 cm. For these calculations we have used the geometry “A” for the two magnets (Magnet 1 and Magnet 2) placed around the Super Mirror. There are 14 pairs of NdFeB stacks in Magnet 1. Their height, thickness and width are $H_1=14$ cm,
T1=2.2 cm and W1=1.858 cm. In Magnet 2 there are 6 pairs of NdFeB stacks. The location of these magnets is specified by the distances DV=14.1 cm, Lx=13.9 cm (both measured from the OZ axis) and L1=2.63 cm, L2=6.193 cm and L3=9.754 cm (measured from the OX axis) (figure 8(A, B)).

Figure 17 (A, B, C, D) The magnetic field is calculated along the OZ axis for three thicknesses T2 of the NdFeB stacks in magnet 2 (figure 17A). The magnetic field uniformity is calculated for the same dimensions along the OZ axis (in figure 17(B)) and along another direction parallel with
OZ (given by X=Y=5 cm) in figure 17 C. The average magnetic field gradient (for a field B0=10 G) is calculated on vertical planes (perpendicular to OZ) in figure 17 (D).

If the center of the Super Mirror is placed at 120 cm upstream from the center of the coils (the origin) then the relative field variation \((B(0, 0, Z) – B0)/B0\) increases to 5% for \(Z= -85 \) cm but is smaller than 0.7% for \(Z> -71 \) cm (the start of the Spin Flipper) (figure 18 (A, B, C)).
Figure 18 (A, B, C, D) The magnetic field B(0, 0, Z) is plotted along the OZ axis in figure 18 (A). The magnetic field uniformity (B-B0)/B0 (%) (with B0 the field in the center of the coils) is plotted along OZ axis for -85 cm < Z < 20 cm (figure 18 B) and -75 cm < Z < 20 cm (figure 18 C). For comparison the magnetic field uniformity in the absence of Magnet2 is provided on the same plots. The magnetic field gradient averaged on vertical planes perpendicular to the axis of the neutron beam is plotted along the OZ axis for -60 cm < Z < 0 cm.

If the center of the Super Mirror is placed at Z=-110 cm then the magnetic field uniformity increases to 35 % for Z=-85 cm and 5.7 % for Z=-70 cm (figure 19 (A, B)). If the thickness of the NdFeB stacks is 3.3 cm < T2 < 3.5 cm then the magnetic field gradient dBy/dy is smaller than 1 mG/cm only if Z>40 cm (for a distance smaller than 40 cm from the center of the coils) (figure 19 (C)).
Figure 19 (A, B, C) The magnetic field along OZ axis is calculated for three thicknesses T2 of the NdFeB stacks in Magnet2 in figure 19 (A). The magnetic field uniformity (B - B0)/B0 (%) is calculated for the same values of T2 and for -85 cm < Z < 20 cm in figure 19(B). The average magnetic field gradient plotted versus Z is presented in figure 19(C).
Conclusion.

A second magnet with only six FeNdB stacks (H2=26 cm) that have the direction of magnetization opposite to the direction of magnetization of the 14 pairs of the NdFeB stacks in Magnet1, decreases the fringe magnetic field along the neutron beam axis and at the same time decreases the magnetic field gradient dBy/dY. If the thickness, width and height of these 14 pairs of magnets in Magnet1 are T1=2 cm, W1=1.858 cm and H1=14 cm then the thickness, width and height of the 6 pairs of magnets in Magnet2 should be T2=2.7±0.1 cm, W2=1.858 cm and H2=25 cm. For a field in the center of the coils equal with 10 Gauss the magnetic field gradient after the Spin Flipper (located between Z=-70 cm and Z=-30 cm) is smaller then 1 (mG/cm) if the exit face of the Super Mirror is located at a distance bigger then 25 cm from the entrance of the Spin Flipper. The center of the Spin Flipper is located at 50 cm distance from the center of the coils. Inside the same region of the neutron beam the magnetic field uniformity is better then 0.7%. Therefore in this case the Spin Flipper can work with maximum efficiency.